AVOIDING THE COLLUSION AMONG THE BIDDERS AND THE AGENT IN SEALED-BID AUCTIONS

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ABSTRACT
We study an auction in which bidders can bribe the auctioneer before they bid and before they know the identity of the winner, with the auctioneer lowering the winner’s bid if the winner is bribed. We show that, in second-price sealed-bid auctions, given the size of the bribe set by the auctioneer, none of the bidders do pay the bribe and every bidder bids his valuation. We also show that the revenue equivalence theorem breaks down when there is bribery because the proposed corruption does not work in the second-price sealed-bid auctions. The first-price and second-price auctions do not yield the same expected revenue to the seller.

Keywords: Auction, Auctioneer, Bribe, Bidders Corruption.

1. INTRODUCTION
In many cases, but not all, a sealed-bid auction has an auctioneer. Sometimes the auctioneer is a third party in the transaction, and sometimes it is an individual who works for the firm awarding the prize and who is given the task of collecting the bids from the bidders. The existence of an agent coming between the seller and the bidders raises the possibility of corruption. One of the ways that corruption occurs is that the auctioneer could look at the submitted bids and then solicit a bribe from the winner after the bids are submitted in exchange for changing the bid in a way that is favorable to the winner. In a standard high-bid auction, this would entail soliciting a bribe in exchange for lowering the winner’s bid down to the second-highest bid. Several existing papers address ex post bribery that occurs after all of the bids are

1 I am indebted to William Neilson, Thomas Jeitschko and Wolfgang Köhler for their helpful comments.

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Another way that corruption occurs is that the auctioneer could solicit bribes from the bidders before the bids are submitted, in exchange for a promise to reduce the bidder’s bid should that bidder be the winner. Koc and Nielson (2005) construct a model to fit this feature. The auction is a first-price sealed bid auction with no reserve price, with the high bidder winning and paying the second-highest bid. Before the bidding, the auctioneer announces the size of the bribe he demands. As many bidders as want to can pay the bribe, and if a bidder who pays the bribe submits the highest bid, the auctioneer lowers the winning bid to the second-highest bid. The high bidder then wins the auction and pays the second-highest bid. In this case, in equilibrium, only bidders with valuations higher than some critical value pay the bribe. Corruption has no effect on either efficiency or the bidders’ expected payoffs when the bidders are symmetric.

Koc and Neilson (2005) shows that in the case where all bidders draw their valuations independently from a single distribution, bidders who have valuations higher than some critical value pay a bribe to the auctioneer, and bidders with low valuations do not. Bidders who pay the bribe bid their own valuations as if they were in a second-price sealed-bid auction, and bidders who do not pay the bribe bid according to the standard equilibrium bid function from the first-price auction. The resulting bid function for all bidders is increasing, and therefore the bidder with the highest value wins the auction, whether he pays the bribe or not, and the auction is efficient. The bidders’ expected equilibrium payoffs are unaffected by corruption. They are neither worse off nor

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2 Lengwiler and Wolfstetter (2000) analyze auctions in which the winning bidder can bribe the auctioneer to change the bid after the auction has ended. Their results are similar to ours, although the results depend on the possibility of the corruption being detected and punished. Menezes and Monteiro (2001) consider a scenario in which there are two bidders and the auctioneer approaches one of them to solicit a bribe in return for changing the bid. The auctioneer can approach either the winner or the loser. Burguet and Perry (2000) study an auction in which one bidder is honest but one is corrupt. Burguet and Che (2004) and Celentani and Ganzuza (2002) study a procurement auction in which the awarding of the contract is based on both the price and the quality of the project, and a corrupt auctioneer can manipulate the quality component in exchange for a bribe.

3 Corruption can also arise through bidding rings, in which the bidders collude to increase their surplus from the seller. See, for example, Graham and Marshall (1987), McAfee and McMillan (1992), and Marshall and Marx (2002). Comte et al. (2000) link the bidding ring literature and the bribery literature with a model of ex post bribery in which the bidders use corruption to enforce collusive behavior.

4 We ignore issues related to the credibility of the auctioneer’s promise, assuming instead that the promise is enforceable. Credibility might occur, for example, if the auctioneer makes this promise repeatedly in auctions over time, so that reputational concerns cause the auctioneer to keep the promise.

better off in terms of the equilibrium expected payoffs. However, there is a transfer of wealth from the seller to the auctioneer. In second-price sealed-bid auctions collusion agreement between the bidders is easier to sustain than in first-price sealed-bid auctions. As Robinson (1985) shows, if there are no problems in coming to agreement among all bidders and abstracting from any concerns about detection, etc., the optimal agreement in a second-price auction is for the designated winner to bid infinitely high while all the other bidders bid zero. No other bidders have any incentive to cheat on this agreement. But in a first-price auction the bidders have to agree that the designated bidder bid a small amount while all the other ones bid zero. In this framework, most of the bidders then have a substantial incentive to cheat on the agreement.\(^6\) However, for the issue of corruption between the auctioneer and the bidders, the scenario is different. In this scenario, corruption takes the following form. The auctioneer approaches the bidders and tells them that if they pay a bribe of a certain amount and if they submit the highest bid, the auctioneer will change their bid so that they only have to pay the second-highest bid. But, in second price auctions bidders have dominant strategy. They bid their values no matter what the other bidders do and pay the second highest bid anyway if they win. Hence, the sealed-bid second price auctions are not vulnerable to the proposed corruption scheme that involves the auctioneer and the winning bidder because they alone cannot change the price. They also need the collaboration of the second highest bidder to pull the price down to the third highest bid.\(^7\) So, the bidders do not accept the offer made by the auctioneer, in other words they do not pay the bribe to the auctioneer. All they do is to play their dominant strategy and bid their value.

This paper is not simply an academic exercise, because \textit{ex ante} bribery has been documented in actual auctions. In their bids for corporate waste-disposal contracts in New York City, Mafia families would sometimes pay bribes for an “undertaker’s look” at the bids of the other bidders before making their own bids.\(^8\) In 1997 a Covington, Kentucky, developer was shown the bids of two competing developers for a $37

\(^6\) Milgrom (1987) develops a similar intuition to argue that repeated second-price auctions are more vulnerable to collusion than repeated first-price auctions.

\(^7\) See Lengwiler and Wolfstetter (2000).

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... million dollar courthouse construction project. In Chelsea, Massachusetts, in the 1980s, the city’s auctioneer was accused of accepting bribes to rig auctions in favor of certain bidders, one time serving as a bidder’s agent in an auction he was running. Lengwiler and Wolfstetter (2000) relay two examples involving German firms which they claim provide evidence of ex post bribery, but we think provide better evidence of ex ante bribery. In one incident, one bidder illegally acquired the application documents of a rival bidder for the Berlin airport construction contract, and in a second incident, Siemens was barred from bidding in public procurement auctions in Singapore for five years because they had bribed an official for information about rival bids. Since the rival bids could be obtained and used before the bidders made their own bids, these could be instances of ex ante bribery.

Finally, we have also been told that auctioneers solicit ex ante bribes for some types of procurement contracts in Turkey. The contracts are auctioned using a standard first-price sealed-bid auction, with the bidder who offers to supply the good at the lowest price winning the auction and supplying the good at that price. Before the bidding starts, the corrupt auctioneer approaches certain bidders with whom he has worked before, and offers to raise their bids to the second-best bid if they win in exchange for a bribe.

This paper analyzes that the seller can avoid the ex ante bribery that occurs before the bids are submitted in sealed-bid auctions by requiring second-price auctions. We show that, in second-price sealed-bid auction, given the size of the bribe set by the auctioneer, none of the bidders do pay the bribe and every bidder bid his valuation. There would be no advantage for them to pay the bribe. As a result, by requiring the auctioneer to run a second-price rather than a first-price auction, the seller can avoid the revenue loss caused by a corrupt auctioneer.

We also show that the revenue equivalence theorem breaks down when there is bribery issue because the proposed corruption does not work in

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9 Crowley, Patrick, Bid Scandal Bill in Trouble, (Cincinnati Enquirer, January 21, 2000).
10 Murphy, Sean P, Chelsea Businessman is Said to Allege Attempted Bribery, (Boston Globe, September 22, 1993).
11 Ingraham (2000) uses empirical methods to study bidder-auctioneer cheating in sealed-bid auctions. Based on statistical properties of the bids, he develops a regression method for analyzing potential cheating of this type. He applies this regression specification to data from the New York City School Construction Authority auctions, and finds evidence that there is cheating between the auctioneer and the bidders.
the second-price sealed-bid auctions. The first-price and second-price auctions do not yield the same expected revenue to the seller.

We proceed as follows: in section 2, we present the game and the notation. Section 3 examines the behavior of the bidders and the auctioneer. Section 4 characterizes the revenue equivalence theorem and shows how it breaks down. Section 5 concludes the discussion.

2. STRUCTURE OF THE GAME
There is a seller of a single good who faces \( n \) risk neutral potential buyers. The seller has hired an auctioneer to run a sealed-bid second-price auction, and pays the auctioneer a fixed wage (as opposed to a commission) in exchange for his services.\(^{12}\) In contrast to the standard second-price auction, the game is supplemented by corruption between the auctioneer and the bidders. The auctioneer approaches every bidder before the auction is held and tells them that if the bidder agrees to pay a bribe of \( \alpha \), and is the highest bidder, he pays the second-highest bid. If the highest bidder did not pay the bribe, he pays his bid. Bribes are collected from all bidders who agreed to pay, even from losing bidders. Consequently, the game is a 3-stage game. In the first stage the auctioneer sets \( \alpha \), in the second stage the bidders decide whether to pay \( \alpha \) independently and simultaneously, and in the third stage the bidders choose their bids.

The bidders’ valuations \( v_1, \ldots, v_n \) are independently and identically drawn from the distribution \( F \) with support \([0,1]\), with a density \( f \), as in the standard symmetric private values model. We assume that the value of the object to the seller is zero and the reserve price is zero. There is no entry fee, making it optimal for all bidders to bid. The seller is passive in this game and we ignore issues related to the detection and punishment of corruption.

We restrict attention to equilibria that survive weak dominance. This rules out preemptive strategies such as one bidder paying the bribe and bidding above 1 while the other bidders do not pay the bribe and bid zero.

\(^{12}\) In the U.S., at least, many auctioneers are paid a commission based on the sales price. Such a payment scheme may reduce the auctioneer’s incentives to solicit bribes, but that issue is left to future research. However, when a firm assigns the task of collecting bids to one of its employees, so that the employee is the auctioneer, that employee is rarely paid a commission.
As is well known, the unique symmetric equilibrium of the second-price auction is the profile of strategies \( (\beta^1, ..., \beta^n) \) such that all \( \beta^i \)'s are equal and all \( \beta^i \)'s are best responses for \( i \) given the strategies of all other bidders. This unique symmetric equilibrium strategy is given by,
\[
b_2(v_i) = v_i.
\]

3. BIDDER AND THE AUCTIONEER BEHAVIOR

In this section we start with the analysis of the behavior of bidders given the size of the bribe, \( \alpha \), set by the auctioneer in first-price auctions. Specifically, we characterize the equilibrium of the subgame that follows the auctioneer’s choice of \( \alpha \). The first task is to find the bids of bidders who do and do not pay the bribe. If a bidder pays the bribe and is the highest bidder, he pays the second highest bid. Therefore, after paying the bribe the bidder essentially participates in a second price auction, and his dominant strategy is to bid his valuation.

**Lemma 1:** Any bidder who pays the bribe bids his valuation, \( v_i \).

Our main result concerns when bidders pay the bribe and when they do not. The next lemma states that bidders use cutoff strategies, that is, for bidder \( i \) there is a valuation \( v_i^* \) such that he pays the bribe when \( v_i \geq v_i^* \) and does not pay the bribe when \( v_i < v_i^* \).

**Lemma 2:** In any equilibrium every bidder uses a cutoff strategy.

**Proof:** See Appendix

However, in a second-price auction no matter what the other bidders do, bidder \( i \) has a dominant strategy: he bids his valuation, \( v_i \). As a matter of fact a bidder, regardless of he pays the bribe or not, bids his valuation. Hence, he does not have incentive to collaborate with the auctioneer and as a result he will not pay any positive amount of bribe.

**Theorem 1:** Given the amount of the bribe \( \alpha \), there exists a unique equilibrium in which bidders with values in \([0,1]\) do not pay the bribe and bid their valuation.

**Proof:** Bidders have dominant strategy in second-price auctions; they bid their valuation. Consider bidder \( i \) with valuation \( v_i \). If he does not pay the bribe he bids \( v_i \) and if he wins he pays the second highest bid, which would be \( v_{(2)} \). Then, his profit would be \( v_i - v_{(2)} \). If he pays the bribe he
bids his value and if he wins he pays $v_{(2)}$. This time his profit would be $v_i - v_{(2)} - \alpha$. As long as bribe is positive bidder $i$ doesn’t pay the bribe.

In the first period the auctioneer chooses the size of the bribe $\alpha$ that a bidder must pay in order to learn the second highest bid if he is the highest bidder. So, the auctioneer aims to maximize his expected revenue by choosing $\alpha$. By Theorem 1, though, for any given $\alpha$, bidders do not accept to pay the bribe to the auctioneer. Therefore, the auctioneer is indifferent about the size of the bribe.

4. BREAKDOWN OF REVENUE EQUIVALENCE THEOREM

As stated earlier the proposed corruption does not work in the second-price sealed-bid auctions and as a matter of fact, the revenue equivalence theorem breaks down. The first-price and second-price auctions do not yield the same expected revenue to the seller.

According to the revenue equivalence theorem, when each of a given number of risk neutral potential bidders of an object has a privately known value independently drawn from a common, strictly increasing distribution, then any auction mechanism in which (i) the highest value bidder always wins the auction, and (ii) any bidder with the lowest feasible value expects zero payoff, yields the same expected revenue to the seller and results in each bidder making the same expected payoff as a function of his value.\(^{13}\)

Koc and Neilson (2004) show that the first-price auction is still efficient and the auction awards the prize to the highest bidder and in equilibrium, bribes are a transfer from the seller to the auctioneer. Although bribery changes the bid functions of some bidders, namely those with sufficiently high valuations, it has no effect on the final allocation of the prize or the welfare of the bidders. From the standard auction theory we know that in the first-price auctions and second-price auctions the expected payoffs of the bidders are identical. As a result, bribery does not affect the expected payoffs of the bidders in two different auctions; they both yield the same expected payoffs to the bidders.

But in terms of the expected revenue of the seller, in the first-price auction with bribery it is the expected value of the second highest value minus the expected revenue of the auctioneer, which is

\[^{13}\text{See Klemperer (1999).}\]
where \( n \) is the number of bidders, \( 1 - F(v^*) \) is the probability that a given bidder pays the bribe, and \( \alpha(v^*) \) is the size of the bribe. In this equation first term is the expected revenue of the seller in the absence of bribery in first or second-price auctions; the second term is the auctioneer’s expected revenue. Koc and Neilson (2004) show that this expected revenue of the auctioneer is strictly positive in first-price auction. On the contrary, the seller’s revenue in the second-price auction with bribery is \( E(v_{(2)}) \) because the second term is zero. Hence, revenue of the seller is strictly greater in the second-price auction than in the first-price auction.

This is an important result that when we introduce the bribery into the model, the revenue equivalence theorem fails.

5. CONCLUSION
In this paper we analyzed a model of bribery in sealed-bid auctions. The bribery involves the auctioneer, who acts as an agent on behalf of the seller, and the bidders. Our results show that, given the size of the bribe set by the auctioneer, none of the bidders do pay the bribe and every bidder bid his valuation if the auction is standard second-price sealed-bid auction. This is because the bidders pay the second highest bid instead of their bids. There would be no advantage for them to pay the bribe. As a result, by requiring the auctioneer to run a second-price rather than a first-price auction, the seller can avoid the revenue loss caused by a corrupt auctioneer.

We also show that the revenue equivalence theorem breaks down when there is bribery issue. The first-price and second-price auctions do not yield the same expected revenue to the seller.
APPENDIX

Proof of Lemma 2: Fix any equilibrium and consider the (right-continuous) cdf, $G_i(b)$, of the highest bid of bidders $j \neq i$. Also let $x_i(b)$ denote the probability of $i$ winning with bid $b$ against the rival bidders employing their equilibrium strategies. (Note that $x_i(b)$ may not equal $G_i(b)$ since a tie may arise at a mass point $b$.) Let $B_c$ be the set of $b$’s for which $G$ is continuous, and let $B_m$ be the set of $b$’s for which $G$ jumps. Then

$$U_{ic}(v) = \int_{b \leq v, b \in B_c} (v-b)dG_i(b) + \sum_{b \leq v, b \in B_m} (v-b)[G_i(b_+)-G_i(b_-)] - \alpha.$$  

$U_{ic}(\cdot)$ is absolutely continuous and can be rewritten as

$$U_{ic}(v) = \int_v^{v'} G_i(s)ds + U_{ic}(v'), \quad (A1)$$

for any $v'$. Now consider

$$U_{in}(v) = \sup_b (v-b)x_i(b).$$

It follows that

$$U_{in}(v) = \max_b (v-b)G_i(b),$$

since $(v-b)G_i(b)$ is an upper envelope of $(v-b)x_i(b)$. One can check that $U_{in}(v)$ is absolutely continuous, that the maximum is well defined (since an upper envelope is upper semicontinuous and the choice can be bound to a compact set without loss of generality), and that $f(b,v) := (v-b)G_i(b)$ is differentiable in $v$ for every $b$ in the equilibrium support. Hence, one can invoke Theorem 2 of Milgrom and Segal to show that

$$U_{in}(v) = \int_v^{v'} G_i(b^*(s))ds + U_{in}(v'), \quad (A2)$$

for $b^*(s) \in \arg\max_b (v-b)x_i(b)$. It follows from (A1) and (A2) that

$$U_{ic}(v) - U_{in}(v) = \int_v^{v'} [G_i(s)-G_i(b^*(s))]ds + [U_{ic}(v') - U_{in}(v')]. \quad (A3)$$
Since $b^*(s) < s$ for almost every $s$, it is clear from (A3) that, whenever $U_{ic}(v') - U_{in}(v') > 0$, it must be that $U_{ic}(v) - U_{in}(v) > 0$ for $v > v'$, which proves that the equilibrium strategy must involve a cutoff strategy with some threshold $v_i^*$.

REFERENCES